

## Mathematical Thinking Skills

CrCrTh650

Fall 2015

Monday, 7-9:45 PM

Online

### **Contact Information**

**Instructor:** Orin C. Davis, Ph.D.

**Office Hours:** Mondays, by appointment only

**Email:** professor.orin.davis+crcrth650@gmail.com

Textbooks:

- 1) Polya, G. (2014). How to solve it: A new aspect of mathematical method. Princeton university press.
- 2) Dixit, A. K. (1991). Thinking strategically: The competitive edge in business, politics, and everyday life. WW Norton & Company.
- 3) Yudkowsky, E. (2015). Rationality: From AI to Zombies. Machine Intelligence Research Institute.

### **Course Description**

This course explores several types of mathematical thinking in the context of number theory, algebra, geometry, and introductory calculus, and relates them to critical and creative thinking skills. Developmental and experiential factors in learning and teaching mathematics are considered, as well as techniques for determining a learner's mathematical abilities and learning styles. Readings, discussion, research, and problem-solving are used to provide a historical context, and to suggest connections with other disciplines. Individual and small-group projects are adapted to student interests. No formal mathematical background beyond high school algebra and geometry is required.

### **Course Overview**

What are the critical and creative thinking skills used by mathematicians to understand concepts and solve problems? How do we develop these skills? In this course, we will begin with an exploration of our own backgrounds in mathematics to understand where our perceptions of and emotions toward mathematics began. We will then focus on different topics in mathematics allowing us to delve into problem solving and reflect on our thinking. Through experiential learning, students will build their understanding of mathematical ideas, highlight their own thinking skills, and then apply them to problem solving. In each class, examples from one or more of calculus, number theory, algebra, geometry, combinatorics, game theory, or other areas of math, will be used as a window into general problem solving and real-world problems. In addition, general problem solving techniques and real-world problems will serve as a basis for gaining insight into how people become critical thinkers in mathematics.

## **Course Objectives**

- Reflect on mathematical development and perceptions, and explain how people learn mathematics.
- Describe key critical and creative thinking skills used in mathematical problem solving.
- Apply examples from calculus, number theory, algebra, and geometry to general problem solving and real-world issues, and vice versa.
- Reflected on the tools, practices, and perspectives from the course that you intend to bring into you specific professional or personal endeavors that you can use in the [Reflective Practitioner's Portfolio](#).

## **Communication**

I will post course materials and announcements on the wiki (<http://crcrth650.wikispaces.umb.edu/>). You are required to check the wiki regularly (at least 4 times per week). *You are responsible for being cognizant of all course announcements.*

Please remember that you are responsible for knowing how to attend the online course punctually and with limited technological glitches. Go to <http://cct.wikispaces.umb.edu/OnlineGuideStudents> for guidelines. *Also note that this course is synchronous and requires all students to attend at the time listed.*

## **School Cancellations**

In the event of a school cancellation, we will resume class as usual on the next scheduled class day. Changes to the class schedule will be discussed during the first class meeting following the cancelled class.

## **Code of Conduct**

The University's Student Code of Conduct ([http://www.umb.edu/life\\_on\\_campus/policies/code](http://www.umb.edu/life_on_campus/policies/code)) exists to maintain and protect an environment conducive to learning. It sets clear standards of respect for members of the University community and their property, as well as laying out the procedures for addressing unacceptable conduct. Students can expect faculty members and the Office of the Dean of Students to look after the welfare of the University community and, at the same time, to take an educational approach in which students violating the Code might learn from their mistakes and understand how their behavior affects others.

It is your own responsibility to understand and abide by UMass's [Student Code of Conduct](#), which includes the university's academic honesty policy. Please note that failing to observe this policy can result in severe penalties ranging from a 0 on an assignment, to failing the course, to being expelled from the university.

## **Accommodations**

Sections 504 and the Americans with Disabilities Act of 1990 offer guidelines for curriculum modifications and adaptations for students with documented disabilities. If applicable, students may obtain adaptation recommendations from the Ross Center (287-7430). The student must present these recommendations to each professor within a reasonable period, preferably by the end of the Drop/Add period.

## Course Requirements and Grading

### Overview of the Grading

Class participation:	25%
Project reflections:	60% (6 entries x 10% each)
Field research:	15 %

***Class participation.*** This is a course rooted in thoughtful discussion and consideration of the material. Ergo, it is incumbent upon you to be prepared *at all times*. Class participation is a major part of the course, and will require that you be active and engaged at each meeting, in addition to occasionally leading class discussions. You will be graded as a function of the depth, thoughtfulness, and activity of your participation. In order to do this, you need to be attentive, and to make substantive statements that go beyond merely [dis]agreeing and/or providing an example of your own experience. You need to explain your line of reasoning and show why the example is relevant. At minimum, constructive commentary requires analysis of the sources to defend a contention. Better is insightful performance that leads the listener to new insights and ideas through complex arguments that are rooted in the interplay and analysis of multiple sources. Often this requires a thesis, a discussion of the caveats to the thesis, and a resolution (synthesis) that indicates when the thesis does and does not apply. A good guideline is to make sure you have spoken substantively at least 2-3 times per class.

***Project Reflections.*** During the second half of most of the class sessions, you will be working on a project that extends from the discussions we had in the first half. At the completion of each project, you will write up what happened in the project and the outcome (1-2 pages, double-spaced, 1" margin, Times New Roman font, well-written), and a reflection about what you learned (1-2 pages, double-spaced, 1" margin, Times New Roman font, well-written). As a guideline, consider that I should know enough about the project from what you wrote that I could replicate it myself without asking you a single question. In your reflections, discuss not only what you learned, but how the readings come to life through the project. Consider what you have learned about the world, about yourself, and about your place in the world.

*Although reflections are not due every week, you are advised to work on these every week so that you don't need to remind yourself what you did in past weeks.*

**REFLECTIONS ARE DUE BY END-OF-DAY (11:59:59 PM, EASTERN TIME) ON THE SUNDAY FOLLOWING THE CLASS IN WHICH THE PROJECT IS COMPLETED.**

**Field Research.** In the Yudkowsky book on rationality, select one section from Book II (e.g., Part E) and one section from Book III (e.g., Part M), and reflect both on each section independently and on the two sections in concord, and address the following:

- 1) What did you learn from them about yourself?
- 2) Based on what you have read, are you a rational thinker? Why or why not?
- 3) Based on what you have read, do you believe most people are rational thinkers? Why or why not?
- 4) Would the world be different if everyone consistently exhibited rational thinking of the kind discussed in these two sections? How so, or how not?

Go out into the world and make some observations about whether you see people exhibit the kind of rationality and/or errors discussed in each of these two sections (one per week). Talk to people (at least 10 people not related to you – preferably strangers, but acquaintances otherwise) and find out whether they are rational thinkers or prone to errors. See if you can get them to see their errors and change their thinking if that’s necessary, and make note of the challenges, successes, and failures in the endeavor.

In your write-up (8-10 pages, double-space, 12-point Times New Roman font, 1” margin), discuss your reflections on items 1-4 above, the procedure for your field research, and the results of your field research. Discuss your successes/failures/challenges pertaining to updating people’s thinking – why did they occur? In your discussion, reflect upon your overall experience doing the project, what you have learned, and how (if at all) you see the world differently on the basis of this assignment.

*Your notes need not be included in your write-up, but you have the option of attaching them if that is instructive (they do not count towards the page limit).*

Two class sessions (9/28, 10/5) have been blocked off to give you the requisite time to embark on this journey. You need not attend class at the appointed time, but you are strongly advised to find a reflection partner with whom to discuss the endeavor.

We will debrief on this assignment in class on 10/12.

**THE WRITE-UP FOR THIS ASSIGNMENT IS DUE BY END OF DAY (11:59:59 PM EASTERN) SUNDAY, 10/11.**

## How to Succeed in This Course

I would like very much for all of you to be successful in this course and to enjoy the learning experience. To that end, these are my recommendations for doing well in my course:

- Keep in mind that this is a graduate-level seminar and that the workload is accordingly higher. Failing to keep up with the readings is not an option in this course, and neither is failing to participate. This holds likewise for preparing the final paper, which you should be working on every week.
- Remember that, although the workload in this course is high, you are going to be going through rapid self-development and learning a great deal. To that end, also make sure that you take space to reflect! You are advised to find a reflection partner with whom you can have a chat outside of class once per week.
- Email me if you have questions, but try to solve the issue yourself, first. When you email me, I will want to know how you have attempted to answer the question so that I may guide you more effectively. Remember, *try to solve it yourself*, and then email me if you cannot do so.
- Remember the following adage: prepare for at least twice as long as the schedule hours of the course. Since there are 3 hours of class per week in this course, you should devote about 6 additional hours per week to preparing for this course. That's less than an hour a day!
- Bring your passion! This is a very exciting subject, and I encourage you to dive in fully.

## Classroom Dynamics

*My teaching style is a bit different from what you have likely encountered previously, so here is some advice and guidelines to orient you.*

***Your success and enjoyment of the course are contingent upon your doing the work. Review the workload and assignments, and drop the course if you are not prepared to do all of it.***

- There are no formal lectures. The entirety of the class is discussion, analysis, and projects.
- As you prepare the class readings, note your observations and questions. Does anything fail to make sense to you? Has something been overlooked by the author(s)? Do you agree or disagree with the authors? Can you refute the perspective(s) presented? Compare the readings to the overview question provided in the syllabus. How do the readings answer (or fail to answer!) the question?
- Many times during the semester, you will find yourself facing ambiguity, and you will be tempted to seek out “the answer the professor is looking for” or “what the professor wants.” Don’t!
  - Instead, use that opportunity to start with an inquiry that is of interest to you, or to provide a piece of information that is a relevant starting point.
  - Remember that this course is meant to be a *learning and growth opportunity* for you, so make the line of discussion, assignment, etc., personally meaningful.
  - The purpose of the ambiguity is to leave room for you to chart your own path!
- During class discussions, you are going to be wrong, you are going to make mistakes, you are going to get your ideas shot down, and you are going to fail. EXPECT THIS.
  - Again, remember that these are *all* opportunities for learning and growth – use them accordingly!
- It is OK to be wrong, it is OK to fail, and it is OK not to have the answer to a question. This will not affect your grade as long as there is evidence that you have prepared the readings (and that will generally be evident in your errors; there is a world of difference between an informed error and an uninformed error!).
  - Remember that I am going to ask you to think beyond the readings, and that will mean hazarding a guess when you don’t actually know the answer. Give it your best shot based on the information you have!
- It is OK to request a moment to think about a response
  - We’ll wait – better that you think for 30 seconds to organize your thoughts than waste our time with an ill-considered response.
- What is *not* acceptable, however, is to be unprepared. If you come to class without preparing the readings *thoroughly*, have the decency and maturity to remain silent unless called upon and accept the penalty for non-participation that day. The only possible exception is if you can back your statement with other information (and think carefully first).
  - *Your uninformed opinion is of no value to those who have done the readings and prepared, and providing it is both rude and a waste of class time. Expect any such rudeness to receive a stern backlash.*
- If you did prepare the readings carefully, then speak up and don’t worry about making errors (honest misconceptions are totally legitimate, and are one of the reasons we discuss in class!)

## Course Calendar and Reading Assignments

9/7 – Introduction to Mathematical Thinking and Strategy

Online lecture posted

Readings: Dixit & Nalebuff, Preface; Yudkowsky, Part A

*No class 9/14*

9/21 – Math in the Classroom

*How do we teach math?*

Readings: Polya, Parts I & II

Serious Introductions (prepare one for class; [instructions here](#))

***CAPS drop deadline: 9/21***

9/28, 10/5 – Field Research (see above)

10/12 – Debrief on Field Research and Strategy

*How do we apply strategy to changing the world around us? Is it just a numbers game?*

Readings: Polya, p. 37-58; Dixit & Nalebuff, Ch. 1

***Students unable to attend must say so in advance to receive an alternate assignment.***

10/19 – Fake Beliefs

*What kinds of errors do we make?*

Readings: Polya, p. 58-75; Yudkowsky, Part B

10/26 – Anticipation

*What is math behind rivalry, and what does that teach us about problem solving?*

Readings: Polya, p. 75-99; Dixit & Nalebuff, Ch. 2-3

***SIGN UP FOR A MEETING WITH THE PROFESSOR NEXT WEEK***

11/2 – Confusion and Uncertainty

*What can we do when the answer isn't obvious?*

Readings: Polya, p. 99-110; Yudkowsky, Part C

11/9 – Thinking Ahead

*How does math help us think ahead, and how does thinking ahead help us with math?*

Readings: Polya, p. 110-129; Dixit & Nalebuff, Ch. 4-5

11/16 – Belief

*How do our beliefs and operating definitions affect our decision processes?*

Readings: Polya, p. 129-149; Dixit & Nalebuff, Ch. 6-7

11/23 – Shooting in the Dark

*Sometimes we have to take a guess to reduce the solution space – when is that a bad idea?*

Readings: Polya, p. 149-162; Yudkowsky, Part D

***11/25 is the last day to drop a course.***

11/30 – Competition and Cooperation

*Does going to extremes actually work? Is win-win a better deal than a Nash Equilibrium?*

Readings: Polya, p. 162-173; Dixit & Nalebuff, Ch. 8-9

12/7 – Preferences and Bargains

*How do we weigh the elements of a problem? What are the tradeoffs we may need to consider?*

Readings: Polya, p. 173-199; Dixit & Nalebuff, Ch. 10-11

12/14 – Incentives

*What motivates people? How does understanding people help us understand the rules of mathematical thinking?*

Readings: Polya, p. 200-225; Dixit & Nalebuff, Ch. 12

*Please remember to do evaluations: <http://bit.ly/CCTEval>*

*The syllabus is subject to change at any time with notice from the professor.*



## References

- Baddeley, A. (2000) The episodic buffer: A new component of working memory? *Trends in Cognitive Sciences*, 4 (11), 417-423.
- Baer, J., & Kaufman, J. C. (2012). *Being creative inside and outside the classroom: How to boost your students' creativity—and your own* (Vol. 2). Springer Science & Business Media.
- Beaty, R. E., Nusbaum, E. C., & Silvia, P. J. (2014). Does insight problem solving predict real-world creativity?. *Psychology of Aesthetics, Creativity, and the Arts*, 8(3), 287.
- Ericsson, K. A. (2014). Why expert performance is special and cannot be extrapolated from studies of performance in the general population: A response to criticisms. *Intelligence*, 45, 81-103.
- Ericsson, K.A. & Charness, N. (1994) Expert performance: Its structure and acquisition. *American Psychologist*, 49, 725-747.
- Ericsson, K. A., Prietula, M. J., & Cokely, E. T. (2007). The making of an expert. *Harvard Business Review*, 85(7/8), 114.
- Geiselman, E.E., Fisher, R.P, MacKinnon, D.P. & Holland, H.L (1986) Enhancement of eyewitness memory with the cognitive interview. *American Journal of Psychology*, 99, 385-401.
- Hofmann, W., Gschwendner, T., Friese, M., Wiers, R. W., & Schmitt, M. (2008). Working memory capacity and self-regulatory behavior: toward an individual differences perspective on behavior determination by automatic versus controlled processes. *Journal of Personality and Social Psychology*, 95(4), 962.
- Lee, C. S., & Theriault, D. J. (2013). The cognitive underpinnings of creative thought: A latent variable analysis exploring the roles of intelligence and working memory in three creative thinking processes. *Intelligence*, 41(5), 306-320.
- Loftus, Elizabeth (1997) Creating false memories, *Scientific American*, (September 1997)
- Plomin, R., Shakeshaft, N. G., McMillan, A., & Trzaskowski, M. (2014). Nature, nurture, and expertise: Response to Ericsson. *Intelligence*, 45, 115-117.
- Raaijmakers, J. G., & Shiffrin, R. M. (1981). Search of associative memory. *Psychological review*, 88(2), 93.
- Sagiv, L., Arieli, S., Goldenberg, J., & Goldschmidt, A. (2010). Structure and freedom in creativity: The interplay between externally imposed structure and personal cognitive style. *Journal of Organizational Behavior*, 31(8), 1086-1110
- Smallwood, J., Obonsawin, M., & Heim, D. (2003). Task unrelated thought: The role of distributed processing. *Consciousness and cognition*, 12(2), 169-189.
- Treisman, A. (1986) Features and objects in visual processing. *Scientific American*, 255 (5), 114-125.
- Wiley, J., & Jarosz, A. F. (2012). Working memory capacity, attentional focus, and problem solving. *Current Directions in Psychological Science*, 21(4), 258-262.